AP® Calculus AB
2004 Sample Student Responses
Form B

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CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

$$\text{Area} = \int_1^{10} (\sqrt{x-1})\,dx = \int_0^9 u^{1/2}\,du = \frac{2}{3}u^{3/2}\Big]_0^9 = 17.999 \text{ units}^2$$

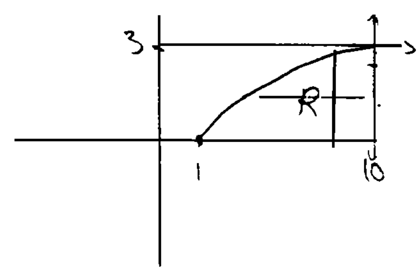
$$\approx 18 \text{ units}^2$$

let $u = x-1$

$\frac{du}{dx} = 1$

$dx = du$

Work for problem 1(b)



Washer's method (strip ⊥)

$R(x) = 3$

$r(x) = 3 - y = 3 - \sqrt{x-1}$

$$V = \pi \int_1^{10} (3)^2 - (3 - \sqrt{x-1})^2\,dx = 212.058 \text{ units}^3$$

Continue problem 1 on page 5.

Work for problem 1(c)

Washer's method (strip ⊥ to x=10)

$y^2 = x-1$

$x = 1+y^2$

$R(y) = 10 - x = 10 - 1 - y^2 = 9 - y^2$

$r(y) = 0$

$V = \pi \int_0^3 (9-y^2)^2 \, dy = 407.150 \text{ units}^3$

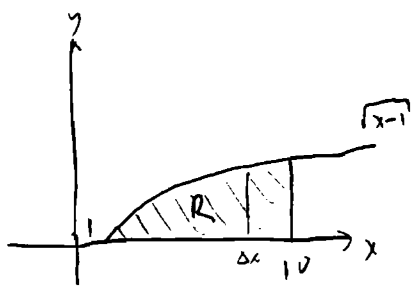
GO ON TO THE NEXT PAGE.

C1

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

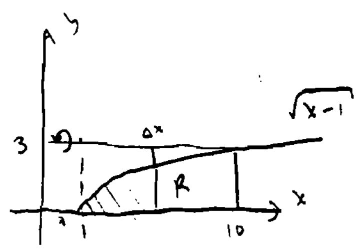
Work for problem 1(a)



$$A_R = \int_1^{10} \sqrt{x-1}\, dx$$

$$A_R = 18.000 \text{ units}^2$$

Work for problem 1(b)



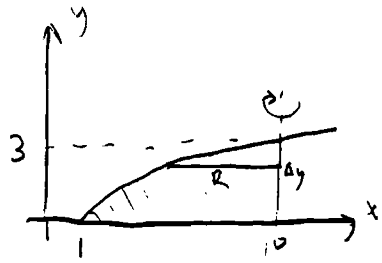
$$V_R = \pi \int_1^{10} (3)^2 - (3 - \sqrt{x-1})^2\, dx$$

$$V_R = 212.058 \text{ units}^3$$

Continue problem 1 on page 5.

 C_2

Work for problem 1(c)



$y = \sqrt{x-1}$

$y^2 = x - 1$

$x = y^2 + 1$

$V_{R_1} = \pi\int_0^3 (y^2+1)^2\,dy$

$V_{R_1} = 218.655 \text{ units}^3$

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Work for problem 2(a)

$R(t) = 5\sqrt{t}\cos\left(\frac{t}{5}\right)$

$R(6) = 4.438$

the derivative of the function is >0 therefore the function is increasing

Work for problem 2(b)

$R'(x) = \frac{2.5\cos(.2x)}{\sqrt{x}} - \sqrt{x}\sin(.2x)$

$R'(6) = -1.913$

increasing at a decreasing rate because $R'(6) < 0$

Continue problem 2 on page 7.

Work for problem 2(c)

$$1000 + \int_0^{31} 5\sqrt{x}\cos\left(\frac{x}{5}\right)dx = 964$$

Work for problem 2(d)

$x = 7.854$ max - absolute

$x = 23.562$ min

$x = 31$ max

$$1000 + \int_0^{7.854} 5\sqrt{x}\cos\left(\frac{x}{5}\right)dx = \boxed{1039}$$

$$1000 + \int_0^{31} 5\sqrt{x}\cos\left(\frac{x}{5}\right)dx = 964$$

1039 mosquitoes is the maximum number for $0 \le - \le 31$

GO ON TO THE NEXT PAGE.

Work for problem 2(a)

$R(6) = 5\sqrt{6}\cos\left(\frac{6}{5}\right) = 4.438 > 0$

the rate of change in the number of mosquitoes is positive for $t=6$
the number of mosquitoes is increasing

Work for problem 2(b)

$R'(t) = \dfrac{5\cos\left(\frac{t}{5}\right)}{2\sqrt{t}} - \sqrt{t}\sin\left(\frac{t}{5}\right)$

$R'(6) = -1.913$

the number of mosquitoes in increasing at adecreasing rate at $t=6$ because $R'(6)$ is negative

Continue problem 2 on page 7.

Work for problem 2(c)

$R(t) = N'(t)$

$N(t)$ is the number of mosquitoes at time t

$N(31) - N(0) = \int_0^{31} R(t)$

$N(31) - 1000 = \int_0^{31} 5\sqrt{t}\cos\left(\frac{t}{5}\right)dt$

$N(31) = -35.665 + 1000 \approx 946$ mosquitoes

there will be 946 mosquitoes at $t = 31$

Work for problem 2(d)

maximum number when $R(t) = 0$

$5\sqrt{t}\cos\left(\frac{t}{5}\right) = 0$

$t = 7.853$ $\quad t = 23.561$

at $t = 7.853$ maximum

$+++ \; 0 \; ----- \; 0 \; +++$

$7.853 \qquad 23.561$

$N(7.853) - N(0) = \int_0^{7.853} R(t)\,dt$

$N(7.853) = 1039.357$

the maximum number of mosquitoes is 1039

GO ON TO THE NEXT PAGE.

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B1

t (minutes)	0	5	10	15	20	25	30	35	40
$v(t)$ (miles per minute)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

Work for problem 3(a)

~~Area = 10(9.2) + 10(7.5)~~ ~~= 10(~~

area = 10(f(5) + f(15) + f(25) + f(35))

= 10(9.2 + 7 + 2.4 + 4.3)

area = 229 miles

$\int_0^{40} v(t)\,dt$ is the total distance traveled between t = 0 and t = 40 minutes

Work for problem 3(b)

a(t) = 0

~~between (...)~~

on the intervals [0,15] and [25,30]

the smallest number of instances the acceleration can equal zero is 2 by MVT and Rolle's Theorem

Continue problem 3 on page 9.

B 2

Work for problem 3(c)

$f'(t) = -\frac{1}{10}\sin t/10 + 3 \cdot \frac{7}{40}\cos 7t/40$

$f'(t) = -1/10 \sin t/10 + 21/40 \cos 7t/40$

$f'(23) = -1/10 \sin 23/10 + 21/40 \cos 161/40$

$f'(23) =$ -.408 miles per minute^2

Work for problem 3(d)

~~average v = $\frac{f(40) - f(0)}{40-0}$~~

~~$= \frac{7.317 - 6}{40}$~~

~~= .033 miles per minute~~

$\frac{1}{40-0}\int_0^{40} 6 + \cos(t/10) + 3\sin(7t/40)\,dt$

average velocity $= \frac{1}{40} \cdot 236.65079$

= 5.916 miles per minute

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

t (minutes)	0	5	10	15	20	25	30	35	40
$v(t)$ (miles per minute)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

Work for problem 3(a)

$$\int_0^{40} v(t)\,dt = \frac{40-0}{4}\left[9.2 + 7 + 2.4 + 4.3\right]$$

$$= \frac{40}{4}[22.9] = 229 \text{ miles}$$

↓ distance plane flies.

Work for problem 3(b)

Acceleration of the plane equals zero where the graph changes concavity. There are 2 such instances one at $t = 10$ min & the other $t \in (25, 30)$.

Continue problem 3 on page 9.

Work for problem 3(c)

$$a(t) = \frac{df}{dt} = \frac{-1}{10}\sin\left(\frac{t}{10}\right) + \frac{21}{40}\cos\left(\frac{7t}{40}\right)$$

$$a(23) = \frac{-1}{10}\sin(2.3) + \frac{21}{40}\cos\left(\frac{161}{40}\right) \approx -0.408 \text{ miles/min}^2.$$

Work for problem 3(d)

$$\text{avg velocity} = \frac{1}{40-0}\int_0^{40} f(t)\,dt = \frac{1}{40}\int_0^{40} 6 + \cos\left(\frac{t}{10}\right) + 3\sin\left(\frac{7t}{40}\right)$$

$$= \frac{1}{40}\left[6t + 10\sin\left(\frac{t}{10}\right) - 3\cos\left(\frac{7t}{40}\right)\left(\frac{40}{7}\right)\right]_0^{40}$$

$$= \frac{1}{40}\left[6t + 10\sin\left(\frac{t}{10}\right) - \frac{120}{7}\cos\left(\frac{7t}{40}\right)\right]_0^{40}$$

$$= \frac{1}{40}\left[240 - 7.568 - 12.924 - \left(-\frac{120}{7}\right)\right]$$

$$= 5.916 \ \frac{\text{miles}}{\text{min}}$$

END OF PART A OF SECTION II

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A,

NO CALCULATOR ALLOWED

CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

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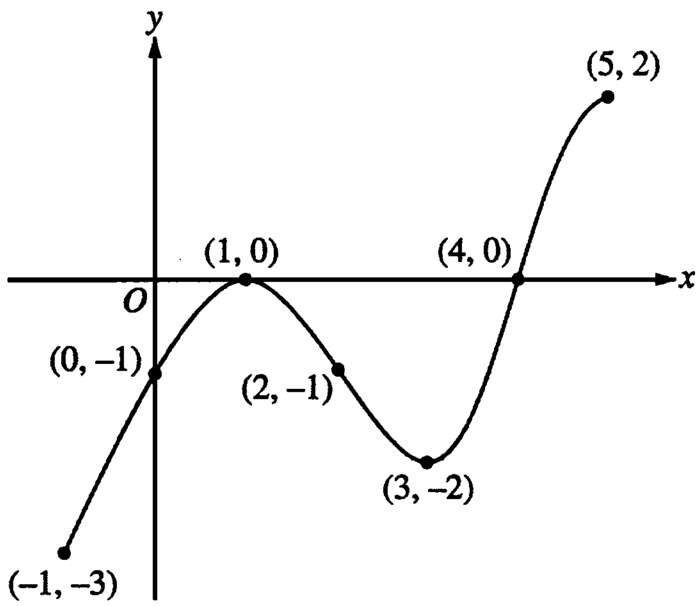


Graph of f'

Work for problem 4(a)

The two points of inflections of f are at $x=1$ and $x=3$.

reason: $f''(x) > 0$ for $x \in (-1, 1)$

$f''(x) < 0$ for $x \in (1, 3)$

$f''(x) > 0$ for $x \in (3, 5)$

Continue problem 4 on page 11.

Work for problem 4(b)

f has an absolute minimum at $x = 4$

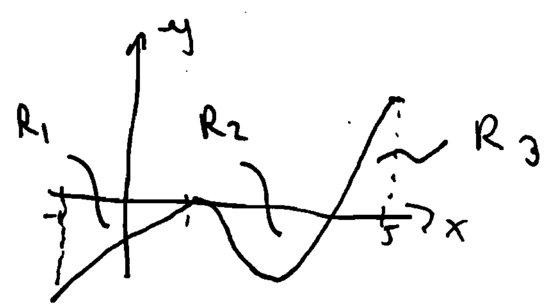
reason: $f'(4) = 0$ and:

x	-1	1	4	5
f'	−	−	+	
f	↘	↘	↗	

↳ abs. min.

f has an absolute maximum at $x = -1$

reason:



$R_1 + R_2 > R_3 \Rightarrow$

$f(-1) > f(5)$

Work for problem 4(c)

$g(x) = x f(x)$

$g'(x) = (x)' f(x) + x f'(x)$

$g'(2) = f(2) + 2f'(2)$

$= 6 + 2 \cdot (-1)$

$= 4$

$g(2) = 2 \cdot f(2)$

$= 12$

$(2, 12)$

$4 = \frac{y - 12}{x - 2}$

$4x - 8 + 12 = y$

$y = 4x + 4$

GO ON TO THE NEXT PAGE.

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CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

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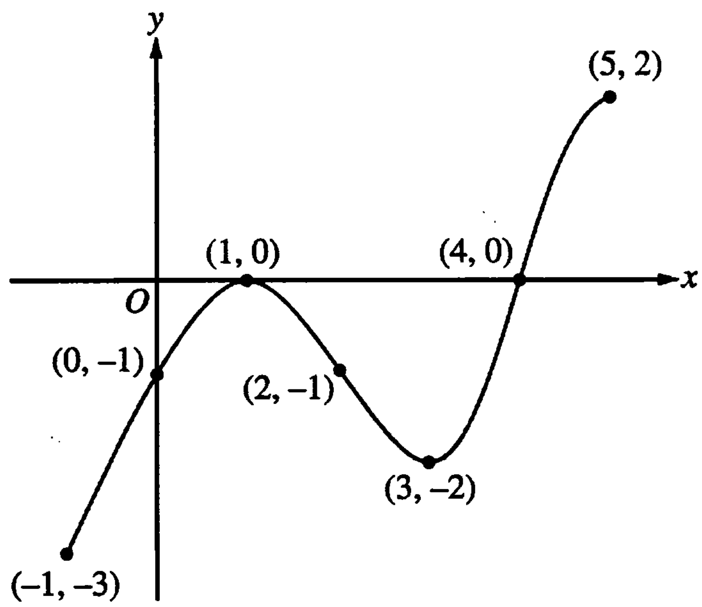

Graph of f'

Work for problem 4(a)

Inflection ⇒ $f''(x)$ changes sign, $f''(x)=0$
⇒ slope of $f'(x)$ changes sign f' slope $= 0$

at $x=1$ slope of $f'(x)$ from +ve to −ve ⇒ inflection
at $x=3$ slope of $f'(x)$ from −ve to +ve ⇒ inflection

Continue problem 4 on page 11.

Work for problem 4(b)

minimum ⇒ $f'(x) = 0$ and $f'(x)$ changes from −ve to +ve

$f'(x) = 0 \Rightarrow x = 4$

x	−1	4	5
$f'(x)$	−		+

⇒ local minimum at $x = 4$ and absolute minimum

maximum ⇒ $f'(x)$ and $f'(x)$ changes from +ve to −ve
but there is no such pt ⇒ check endpoints

The decrease from $x = -1$ to $x = 4$ is more than increase from $x = 4$ to $x = 5$

⇒ $f(5) < f(-1)$ ⇒ abs. max at $x = 5$

Work for problem 4(c)

$g(x) = x f(x)$

$g'(x) = f(x) + x f'(x)$

$g'(2) = f(2) + 2f'(2) = 6 + 2(-1) = 4$

⇒ $g(2) = 2f(2) = 2(6) = 12$

⇒ $y - 12 = 4(x - 2)$

eq of tan $y = 4x + 4$

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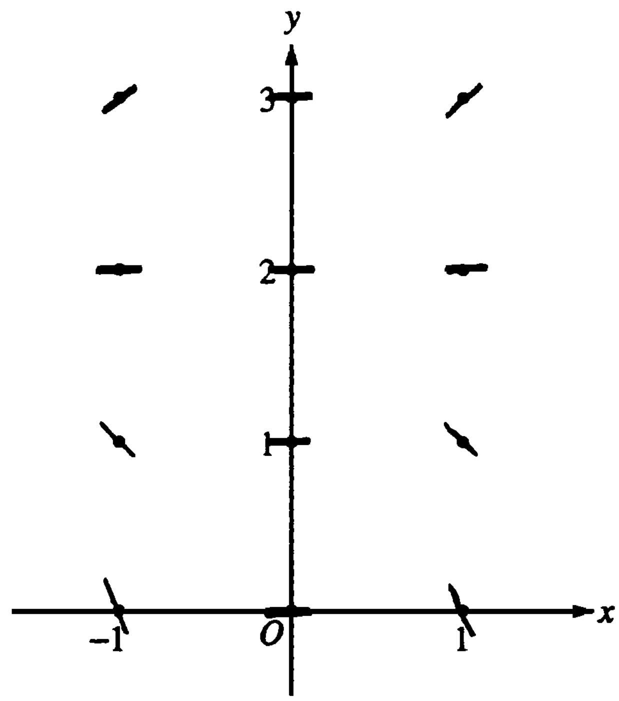
(A)

Work for problem 5(a)



Work for problem 5(b)

x^4 is always positive $\Rightarrow \frac{dy}{dx} < 0$ iff $y < 2 \wedge x \neq 0$

$\therefore$ the negative slope's where $y < 2$ and $x \neq 0$.

~~$\Rightarrow$ the negative slopes~~ become greater in magnitude as $|x|$ become greater and $|y-2|$ become greater

Continue problem 5 on page 13.

A2

NO CALCULATOR ALLOWED

Work for problem 5(c)

$$\frac{dy}{dx} = x^4(y-2) \Rightarrow \frac{dy}{(y-2)} = x^4\,dx \Rightarrow \int \frac{dy}{(y-2)} = \int x^4\,dx$$

$$\Rightarrow \ln|y-2| = \frac{x^5}{5} + c_1 \Rightarrow y-2 = e^{\frac{x^5}{5}+c_1} \Rightarrow y = Ce^{x^5/5} + 2$$

$$f(0) = 0 \Rightarrow 0 = Ce^0 + 2 \Rightarrow C = -2$$

$$\therefore\ y = -2e^{\frac{x^5}{5}} + 2$$

GO ON TO THE NEXT PAGE.

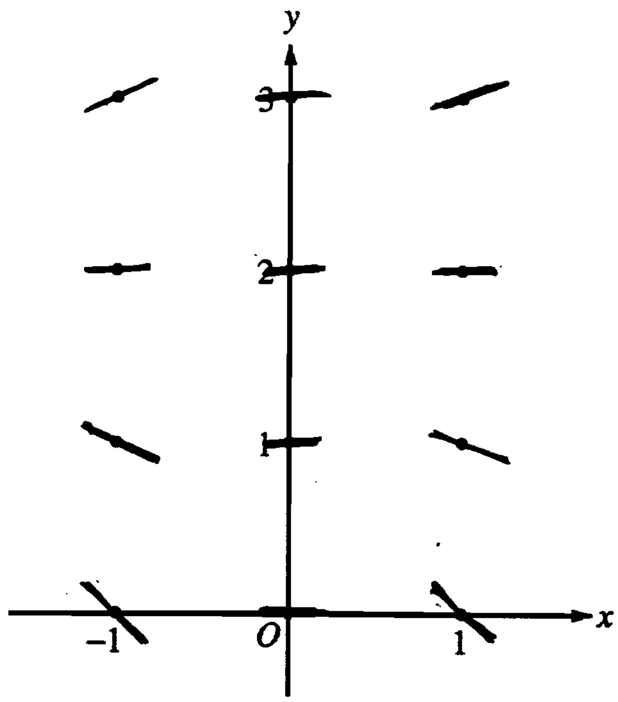
D1

NO CALCULATOR ALLOWED

Work for problem 5(a)

$\frac{dy}{dx} = x^4(y-2)$

(x,y)	$\frac{dy}{dx}$
(0,0)	0
(0,1)	0
(0,2)	0
(0,3)	0
(1,0)	−2
(1,1)	−1
(1,2)	0
(1,3)	1



(x,y)	$\frac{dy}{dx}$
(−1,0)	−2
(−1,1)	−1
(−1,2)	0
(−1,3)	1

Work for problem 5(b)

There are four points for which the slopes are negative in Part (a)

They are (1,0) (1,1) (−1,0) and (−1,1)

If the slope ($\frac{dy}{dx}$) is negative, that means the graph of y is decreasing at these four points

Continue problem 5 on page 13.

D2

NO CALCULATOR ALLOWED

Work for problem 5(c)

$$\frac{dy}{dx} = x^4(y-2)$$

$$x^4\,dx = \frac{dy}{y-2}$$

$$\int x^4\,dx = \int \frac{dy}{y-2}$$

$$\frac{x^5}{5} + C_1 = \ln|y-2| + C_2.$$

$$\ln|y-2| = \frac{x^5}{5} + C$$

$$y-2 = e^{\frac{x^5}{5}+C} = e^{\frac{x^5}{5}} \cdot e^C$$

$$= Ae^{\frac{x^5}{5}}$$

$$f(x) = y = Ae^{\frac{x^5}{5}} + 2 \qquad f(0) = 0$$

$$f(0) = Ae^{\frac{0^5}{5}} + 2 = 0.$$

$$Ae^0 + 2 = 0$$

$$A + 2 = 0$$

$$A = -2$$

$$\therefore \boxed{y = -2e^{\frac{x^5}{5}}}$$

GO ON TO THE NEXT PAGE.

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B

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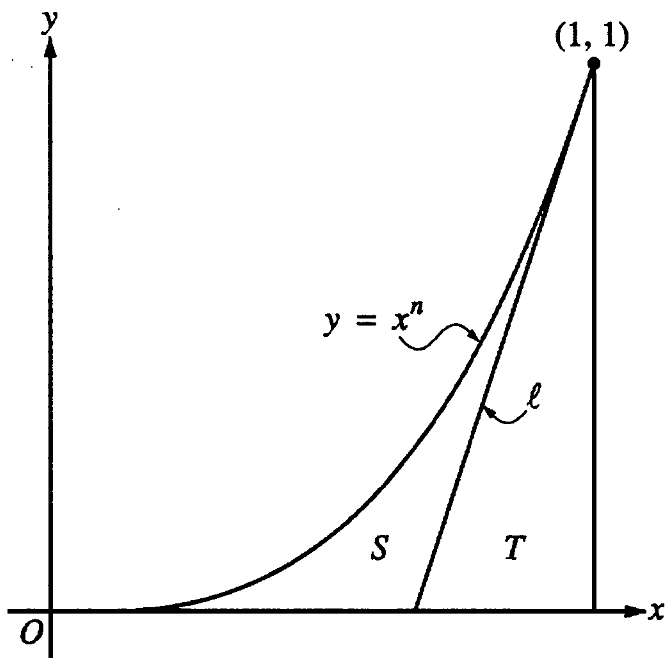

Work for problem 6(a)

$$\int_0^1 x^n\,dx = \left(\frac{x^{n+1}}{n+1}\right)_0^1 = \frac{1^{n+1}}{n+1} - \cancel{\frac{0^{n+1}}{n+1}}$$

0

$$= \frac{1}{n+1}$$

Work for problem 6(b)

$y = x^n$

$y' = nx^{n-1}$

$y'(1) = n\,1^{n-1}$

$= n$

i) the equation of the tangent line will be

$y - 1 = n(x-1)$

$\boxed{y = nx - n + 1}$

in order to get base of T,

$y = nx - n + 1$

$0 = nx - n + 1$

$\frac{n-1}{n} = x$

$$A_{\text{of } T} = \frac{1 \times \left(1 - \frac{n-1}{n}\right)}{2}$$

$$= \frac{\frac{n-n+1}{n}}{2}$$

$$\boxed{= \frac{1}{2n}}$$

Continue problem 6 on page 15.

Work for problem 6(c)

$A_S = \int_0^1 x^n\,dx - A_T$

from (a) & (b) we know $\int_0^1 x^n\,dx$ & A_T

$\therefore A_S = \frac{1}{n+1} - \frac{1}{2n}$

$= \frac{2n-n-1}{2n(n+1)} = \frac{+n-1}{2n(n+1)}$ Answer

$A_S = \frac{(n-1)}{2n^2+2n}$

$A'_S = \frac{2n^2+2n-(n-1)(4n+2)}{(2n^2+2n)^2} = 0$

$2n^2+2n-(4n^2-2n-2) = 0$

$n^2+n-2n^2+n+1=0$

$-n^2+2n+1=0.$

by quadratic formula.

$\frac{-2\pm\sqrt{4+4}}{-2} = \frac{-2\pm 2\sqrt{2}}{-2} = 1\pm\sqrt{2}$

$A_S(1+\sqrt{2}) = \frac{\sqrt{2}}{2(1+2\sqrt{2}+2)+2+2\sqrt{2}}$

$= \frac{\sqrt{2}}{6+2+2\sqrt{2}+4\sqrt{2}}$

$= \frac{\sqrt{2}}{8+6\sqrt{2}}$

$A_S(1-\sqrt{2}) = \frac{-\sqrt{2}}{2(1-2\sqrt{2}+2)+2(1-\sqrt{2})}$

$= \frac{-\sqrt{2}}{6-2\sqrt{2}+2-2\sqrt{2}}$

$= \frac{-\sqrt{2}}{8-2\sqrt{2}}$

Answer

$n = 1+\sqrt{2}$

END OF EXAMINATION

THE FOLLOWING INSTRUCTIONS APPLY TO THE BACK COVER OF THIS SECTION II BOOKLET.

- MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE BACK OF THIS SECTION II BOOKLET.
- CHECK TO SEE THAT YOUR AP NUMBER APPEARS IN THE BOX(ES) ON THE BACK COVER.
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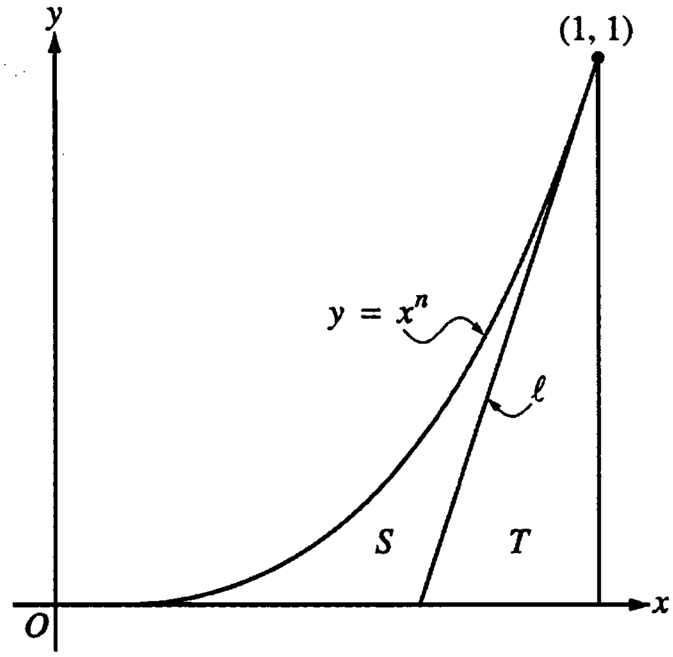
D1

NO CALCULATOR ALLOWED



Work for problem 6(a)

$$\int_0^1 x^n\,dx = \frac{x^{n+1}}{n+1}\Big]_0^1 = \left(\frac{(1)^{n+1}}{n+1}\right) - \left(\frac{(0)^{n+1}}{n+1}\right)$$

$$= \frac{1}{(n+1)} - \frac{0}{n+1} = \frac{1}{n+1} \text{ units}^2$$

Work for problem 6(b)

$$y = x^n$$

$$\frac{dy}{dx} = nx^{n-1}$$

$$\frac{dy}{dx}\Big|_1 = n$$

eqn. of line ℓ:

$$y - y_0 = m(x - x_0)$$

~~y - 1 =~~

$$y - 1 = n(x - 1)$$

$$y - 1 = nx - n$$

$$y = nx - n + 1$$

x-intercept ⇒ ~~y = 0 = nx + 1 = 0~~ → $nx - n + 1 = 0$

$y = 0$ → ~~n(x - 1) + 1 = 0~~ ~~n =~~

$$nx = n - 1$$

$$x = \frac{n-1}{n}$$

$$\text{Area} = \frac{1}{2}(1)\left(1 - \frac{n-1}{n}\right)$$

Continue problem 6 on page 15.

$$\text{Area} = \frac{1}{2}(1)\left(\frac{n-n-1}{n}\right) = \frac{1}{2}(1)\left(1 - \frac{1}{n}\right) = \frac{1}{2}(1)\left(\frac{1}{n}\right) = \frac{1}{2n} \text{ units}^2$$

D2

NO CALCULATOR ALLOWED

Work for problem 6(c)

Area of S $= \int_0^1 x^n \, dx$ − Area of T

$= \frac{1}{n+1} - \frac{1}{2n}$

$= \frac{(2n) - (n+1)}{2n(n+1)}$

~~= n+1~~ $\frac{n-1}{2n^2+2n}$ units^2

maximum : $S'(n) = 0$

$\Rightarrow$ $(1)(2n^2+2n) - (4n+2)(n-1) = 0$

$2n^2+2n - (4n^2 - 4n + 2n - 2) = 0$

$2n^2 + 2n - 4n^2 + 4n - 2n - 2 = 0$

$-2n^2 + 4n - 2 = 0$

$n^2 - 2n + 1 = 0$

$(n-1)^2 = 0 \Rightarrow n = 1$ will maximize the area.

END OF EXAMINATION

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